Automorphism of a group

Def An isomorphic mapping of a group G onto itself is called an automorphism of G.

Thus $f : G \xrightarrow[\text{One-one}]{\text{onto}} G$ is an automorphism of G if

$$f(ab) = f(a)f(b) \ \forall \ a, b \in G.$$

Show that the mapping $f : I \to I$ such that $f(x) = -x \ \forall x \in I$ is an automorphism of the additive group of integers I.

Sol^n The mapping f is One-One onto.
Let x_1, x_2 be any two elements of I
Then

$$f(x_1 + x_2) = -(x_1 + x_2)$$

$$= (-x_1) + (-x_2)$$

$$= f(x_1) + f(x_2)$$

Hence f is an automorphism of I.

9-12-20 Part III PAPER-6 Page No.: Date: / /

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Date 9-12-20

Page No.:
Date: / /

Let G be a group, then the identity map $I: G \to G$ s.t. $I(x) = x$ is trivially an automorphism of G & In fact it is sometimes called trivial automorphism.

Let Z = Group of integer under addition then $f: Z \to Z$, s.t.

$$f(n) = -n$$

is an automorphism as $f(n) = f(m)$
$\Rightarrow -n = -m \Rightarrow n = m \Rightarrow f$ is 1-1
Again since for any $n \in Z$.

$f(-n) = n$ we find f is Onto
Now $f(n+m) = -(n+m) = -n-m$

$$= f(n) + f(m)$$

Shows f is a homomorphism and hence automorphism.